MATH 2B/5B Prep: Limits at Infinity

1. Calculate the limit $\lim_{x\to\infty} \frac{x^2+x^3}{2x-1}$.

Solution: Since this is a rational function the first step is to divide the numerator and denominator by the largest power of x that appears, here that is x^3. This gives

$$\lim_{x\to\infty} \frac{x^2+x^3}{2x-1} \cdot \frac{1/x^3}{1/x^3} = \lim_{x\to\infty} \frac{1/x+1}{2/x^2-1/x^3}$$

Then as x goes to infinity we see that the numerator goes to 1 while the denominator goes to 0, meaning that

$$\lim_{x\to\infty} \frac{x^2+x^3}{2x-1} = \infty$$

2. Evaluate $\lim_{x\to\infty} \frac{x(x+1)(2x+1)}{6x^3}$.

Solution: Again the first step is to divide by the largest power appearing in the fraction. Notice that if we multiplied out the numerator the largest power would be x^3 which is also the power of the denominator. Then we get

$$\lim_{x\to\infty} \frac{x(x+1)(2x+1)}{6x^3} \cdot \frac{1/x^3}{1/x^3} = \lim_{x\to\infty} \frac{(1+1/x)(2+1/x)}{6} = \frac{(1+0)(2+0)}{6} = \frac{2}{6} = \frac{1}{3}$$

3. Find the limit $\lim_{x\to\infty} \frac{\sin(x)}{\sqrt{x}}$. Hint: Consider the Squeeze Theorem.

Solution: Since $-1 \leq \sin(x) \leq 1$ we have that

$$\frac{-1}{\sqrt{x}} \leq \frac{\sin(x)}{\sqrt{x}} \leq \frac{1}{\sqrt{x}}$$

Now

$$\lim_{x\to\infty} \frac{-1}{\sqrt{x}} = 0$$
$$\lim_{x\to\infty} \frac{1}{\sqrt{x}} = 0$$

So by Squeeze Theorem $\lim_{x\to\infty} \frac{\sin(x)}{\sqrt{x}} = 0$ as well.

4. Evaluate $\lim_{x\to\infty} \frac{x}{e^x}$.

Solution: First we see that both the numerator and denominator go to infinity as x goes to infinity, so this is an $\frac{\infty}{\infty}$ indeterminate form and we can use L'Hospital's Rule. Remember that when using the rule we take the derivative of the numerator and denominator separately and don't use quotient rule.

$$\lim_{x\to\infty} \frac{x}{e^x} = \lim_{x\to\infty} \frac{1}{e^x} = 0$$

since the numerator is constant and the denominator goes to infinity.